The Role of Existential Graphs in Peirce's Philosophy

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Abstract. Nowadays, Peirce is mostly recognized as the founder of pragmatism and for his extensive theory of signs. Interestingly, in contrast to the contemporary estimation of his work, Peirce himself considered his system of existential graphs as the 'luckiest find of my career'. This paper aims to clarify why Peirce placed his existential graphs into the very heart of his philosophy. Moreover, the design of the graphs as diagrammatic reasoning system, as well as the design of the transformation rules, can be explained with Peirce's purpose in the development of his graphs.

> Diagrammatic reasoning is the only really fertile reasoning. If logicians would only embrace this method, we should no longer see attempts to base their science on the fragile foundations of metaphysics or a psychology not based on logical theory; and there would soon be such an advance in logic that every science would feel the benefit of it.
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> Peirce, Prolegomena to an Apology For Pragmaticism, 1906

1 Introduction

Among philosophers, Peirce is in the first place recognized as the founder of 'pragmatism' (or 'pragmaticism', as Peirce later called his theory), and as a scientist who has elaborated the probably most extensive theory of signs, i.e., semiotics. But the system of existential graphs is neither in philosophy, nor in mathematics or logic, very much acknowledged or even appreciated. Interestingly, in contrast to the contemporary estimation of his work, Peirce himself considered his development of existential graphs as his 'luckiest find of my career', and he called them his 'chef d'oeuvre'. In a letter to William James, he says that EGs are the 'logic of future'. In fact, after he started working with EGs, he spent the remainder of his life with the elaboration of this system. Mary Keeler writes in [Kee] that '*he produces his most intensive theoretical work, which includes the Existential Graphs, during the last 10 years of his life (40.000 pages, or* *nearly half of the whole collection [100.000 unpublished pages which are achieved in the the Houghton Library at Harvard]).*'

This paper attempts to explain why Peirce places his existential graphs into the center of his philosophy, and from this elaboration we will moreover obtain good reasons why Peirce designed the existential graphs the way he did.

2 Foundations of Knowledge and Reasoning

The overall goal of Peirce's philosophy are the foundations of reasoning and knowledge. Hookway, who has worked extensively with Peirce's manuscripts, writes in [Hoo85]: '*Inspired by Kant, he devoted his live to providing foundations for knowledge and, in the course of doing so, he brought together a number of different philosophical doctrines*', and Mary Keeler says in [Kee] that '*generally, his life's work can be seen as a struggle to build the philosophical perspective needed to examine how intellectual growth occurs.*'

Peirce's semiotics and his theory of pragmaticism can be seen as two important facets of his theory of reasoning. Pragmaticism is not addressed by this paper, thus I let other authors describe the relationship between pragmatism and reasoning. The editors of the collected papers summarize in the introduction of Volume 5 (Pragmatism and Pragmaticism) this relationship as follows: '*Pragmatism is conceived to be a method in logic rather than a principle of metaphysics. It provides a maxim which determines the admissibility of explanatory hypotheses.*' Similarly, Dipert writes in [Dip04] that '*the penultimate goal of thought is to have correct representations of the world, and these are ultimately grounded for the pragmatist in the goal of effective action in the world.*' I.e., as Dipert writes, pragmaticism answers the question *why* to think logically.

2.1 Logic and Semotics

More important to us is the relationship between semiotics and reasoning. For Peirce, semiotics is not a mere metatheory of linguistics, he is interested in what sense signs are involved in reasoning. Already in 1868, in a publication titled 'Questions concerning certain Faculties Claimed for Man', he addresses the question whether reasoning which does not use signs is possible, and he comes to the conclusion that '*all thought, therefore, must necessarily be in signs*' (whole article: 5.213-5.263, quotation: 5.252[1]). Particularly, the main types of signs, i.e. icons, indices, and symbols (see [Dau04, Shi02] for a discussion), are needed in reasoning. In 5.243, Peirce claims that these '*three kinds of signs [...] are all indispensable in all reasoning.*' It is not only reasoning which has to be in signs. Pape summarizes the following fundamental principle which underlies

[1] I adopt the usual convention to refer to the colected papers [HB35]. I.e., 5.213-5.263 refers to the fifth book of [HB35], paragraphs 213–263.

Peirce understanding of semiotics:[2] '*All intellectual or sensory experience – no matter of which pre-linguistic or pre-conscious level it is – can be generalized in a way that it can be interpreted in a universal representation.*'

In his speculative grammar (2.105–2.444), Peirce's elaborates that the growth of knowledge is condensed in the change and growth of the meaning of signs. In 2.222, he writes: '*For every symbol is a living thing, in a very strict sense that is no mere figure of speech. The body of the symbol changes slowly, but its meaning inevitably grows, incorporates new elements and throws off old ones.*' In this understanding, semiotics is more than a formal theory of signs: It is a theory of meaning as well. Moreover, to investigate the laws of reasoning is to investigate the relationships between the signs reasoning is based on. Thus a theory of reasoning and the emergence of knowledge has to be a theory of signs. In 1.444, Peirce summarizes the relationship between logic, reasoning and semiotic as follows: '*The term "logic" [...] in its broader sense, it is the science of the necessary laws of thought, or, still better (thought always taking place by means of signs), it is general semeiotic, treating not merely of truth, but also of the general conditions of signs being signs [...], also of the laws of the evolution of thought.*' Due to this broad understanding of semiotics and logic, these two research fields investigate reasoning from different perspectives, but they are essentially the same. So Peirce starts the second chapter of his speculative grammar with the conclusion that '*logic, in its general sense, is, as I believe I have shown, only another name for semiotic*' (2.227).

2.2 Necessary Reasoning

In the following, we will investigate Peirce's theory of logic and reasoning. I start this scrutiny with two quotations from Peirce, both taken from 'Book II: Existential graphs' of the collected papers, in which he elaborates his understanding of logic and so-called *necessary reasoning*. In 4.431, Peirce writes:

> But what are our assertions to be about? The answer must be that they are to be about an arbitrarily hypothetical universe, a creation of a mind. For it is necessary reasoning alone that we intend to study; and the necessity of such reasoning consists in this, that not only does the conclusion happen to be true of a pre-determinate universe, but will be true, so long as the premises are true, howsoever the universe may subsequently turn out to be determined. Physical necessity consists in the fact that whatever may happen will conform to a law of nature; and logical necessity, which is what we have here to deal with, consists of something being determinately true of a universe not entirely determinate as to what is true, and thus not existent.

[2] The original German quotation is: 'Alle intellektuelle und sinnliche Erfahrung – gleich welcher vorsprachlichen oder vorbewußten Stufe – kann so verallgemeinert werden, daß sie in einer universalen Darstellung interpretierbar ist.'

In 4.477, we find:

> The purpose of logic is attained by any single passage from a premiss to a conclusion, as long as it does not at once happen that the premiss is true while the conclusion is false. But reasoning proceeds upon a rule, and an inference is not necessary, unless the rule be such that in every case the fact stated in the premiss and the fact stated in the conclusion are so related that either the premiss will be false or the conclusion will be true. (Or both, of course. "Either A or B" does not properly exclude "both A and B.") Even then, the reasoning may not be logical, because the rule may involve matter of fact, so that the reasoner cannot have sufficient ground to be absolutely certain that it will not sometimes fail. The inference is only logical if the reasoner can be mathematically certain of the excellence of his rule of reasoning; and in the case of necessary reasoning he must be mathematically certain that in every state of things whatsoever, whether now or a million years hence, whether here or in the farthest fixed star, such a premiss and such a conclusion will never be, the former true and the latter false.

The main point in both quotations is that Peirce's emphasizes to investigate *necessary* reasoning, and he elaborates his understanding of necessity in reasoning. First of all, we see that a necessary implication is an implication which can never lead from a true premise to a false conclusion. This can be expressed by different logical connectives: In the second quotation, he explicates a necessary inference like a truth-table (to adopt a term from contemporary propositional logic) with the operators 'not' and 'or'. In another place, he writes: '*A leading principle of inference which can lead from a true premiss to a false conclusion is insofar bad; but insofar as it can only lead either from a false premiss or to a true conclusion, it is satisfactory; and whether it leads from false to false, from true to true, or from false to true, it is equally satisfactory*'; thus in this quotation he provides the truth-table for the syntactical expression $a \to b$. A necessary implication corresponds to the material implication as it is understood in classical propositional logic, that is, as an implication which can be expressed in the following different ways:

$$a \to b \quad \sim \quad \neg(a \land \neg b) \quad \sim \quad \neg a \lor b$$

The truth of a necessary implication does not depend on the actual *facts* expressed in its premise and conclusion, but only on its *form*. An implication can be a 'physical necessity' if it is true due to physical laws, but here are still facts involved: Only if an implication is true in an '*arbitrarily hypothetical universe, a creation of a mind*', i.e. it is true in '*every state of things whatsoever*', then it is a necessary implication. Moreover, considering hypothetical universes fits very well into the contemporary tarski-style approach to logic and model-theory, where the different states of things, the different universes of discourses are mathematically captured by (usually relational) models, and an implication is true (a better word in mathematical logic would be 'valid') if it holds in every model.

Peirce had been a mathematician on its own, having a deep respect for mathematics and their kind of reasoning (in 4.235, he appraises the mathematicians as follows: '*Mathematicians alone reason with great subtlety and great precision.*' It is important to understand the role mathematics plays among the sciences in Peirce's philosophy. In 4.232, he explains his view what the '*true essence of mathematics*' is: '*For all modern mathematicians agree with Plato and Aristotle that mathematics deals exclusively with hypothetical states of things, and asserts no matter of fact whatever; and further, that it is thus alone that the necessity of its conclusions is to be explained.*' Dealing not with actual facts, but exclusively with hypothetical states of things is the essence of mathematics, not shared with any other science. In his Cambridge lectures ([Pei92]), lecture one, Peirce provides a classification of science which is based on their level of abstraction: A science is placed above a second one if the second science adopts the principles of the first science, but not vice versa. Mathematics is the science at the top of this classification '*for this irrefutable reason, that it is the only of the sciences which does not concern itself to inquiry what the actual facts are, but studies hypotheses exclusively.*' In this sense, even philosophy is more concrete than mathematics, as it is '*a search for the real truth*' and as '*it consequently draws upon experience for premises and not merely, like mathematics, for suggestions.*' As mathematics is the only science which does not deal with facts, but with hypothetical universes, it is clear why Peirce identifies necessary and mathematical reasoning. He explicates this very clearly in his lectures when he says that '*all necessary reasoning is strictly speaking mathematical reasoning*'.

2.3 The Self-Correcting Property of Reason

In 4.425–4.429, Peirce makes clear that mathematical reasoning is by no means a mere application of some static inference rules. He starts his observations with an examination of the syllogisms of Aristotle from which he says that the '*ordinary treatises on logic [...] pretend that ordinary syllogism explains the reasoning of mathematics. But if this statement is examined, it will be found that it represents transformations of statements to be made that are not reduced to strict syllogistic form; and on examination it will be found that it is precisely in these transformations that the whole gist of the reasoning lies.*' When Peirce wrote these sentences, after an absolute dominance of syllogism which lasted for more than two thousand years, new approaches to a formal theory of logic and necessary reasoning emerged. Peirce, as a researcher in this field, was of course aware of these approaches. In these paragraphs, he mentions Schröder, Dedekind, and his own systems. In other places, he discusses (and extends) Boole's approach to a large extend. But none of these approaches are comprehensive enough or even sufficient to capture the whole realm of reasoning, that is '*that the soul of the reasoning has even here not been caught in the logical net*' (4.426). And even more explicit, in the beginning of the 4.425, he writes:

> But hitherto nobody has succeeded in giving a thoroughly satisfactory logical analysis of the reasoning of mathematics.[...] yet nobody has

> drawn up a complete list of such rules covering all mathematical inferences. It is true that mathematics has its calculus which solves problems by rules which are fully proved; but,[...] every considerable step in mathematics is performed in other ways.

We see that there is no comprehensive theory of mathematical reasoning. Moreover, Peirce is aware that mathematician are human beings which may make mistakes in their reasoning. In [Pei92], lecture 4, he writes: 'Theoretically, *I grant you, there is no possibility of error in necessary reasoning. But to speak thus 'theoretically', it is to use language in a Pickwickian sense. In practice and in fact, mathematics is not exempt from the liability to error that affects everything that man does*' (emphasis by Peirce). In the light of these observations, the question arises why Peirce had so much trust in the reliability and certainty of mathematical reasoning.

The clue is '*this marvellous self-correcting property of Reason*', as Peirce says in [Pei92]. Reasoning is a conscious process which in turn can be subject of inspection, criticism, or reasoning itself. This ability of self-criticism[3] is crucial to call any inference-process 'reasoning'; it distinguishes reasoning from a a mere, mechanical application of inference rules to obtain conclusions from to premises. In 1.606 (a work titled 'ideals of conduct'), Peirce expresses this point: '*For reasoning is essentially thought that is under self-control. [...] You will nevertheless remark, without difficulty, that a person who draws a rational conclusion, not only thinks it to be true, but thinks that similar reasoning would be just in every analogous case. If he fails to think this, the inference is not to be called reasoning.*' The ability of self-control includes the ability of self-criticism: '*But in the case of reasoning an inference which self-criticism disapproves is always instantly annulled, because there is no difficulty in doing this*' (1.609).

The ability of self-criticism implies an important consequence. The conclusions of some train of reasoning are not simply granted fortrue: They are observed and verified. The verification of the truth of the conclusion may fail. In this case, the reasoning has to be corrected. The correction not only concerns the result of the reasoning: The assumptions the reasoning started with, even if they had been taken for true so far, may be corrected, too. In [Pei92], Peirce's writes: '*I can think of, namely, that reasoning tends to correct itself, and the more so the more wisely its plan is laid. Nay, it not only corrects its conclusions, it even corrects its premises.*'

[3] Self-reference and self-criticism are based on a specific kind of abstraction, a shift of the observing level from the use of (linguistic) items to their observation. It should be noted that it is this shift of levels which underlies Peirce's already mentioned conception of *hypostatic abstraction*, where a former collection of items is considered to be a new, singular item of its own.

2.4 Deduction, Induction, and Abduction

Peirce distinguishes between three modes of reasoning. *Induction* concludes its conclusion from a sufficient large amount of facts; that is, the conclusion is an approximate proposition which generalizes and explains these facts. This is the mode of inquiry which occurs as main mode of reasoning in sciences which are based on experiments. Induction leads to truth in the long run of experience.

Deduction concludes its conclusion not from the content of the premises, but from the form of the argumentation. It may happen that the conclusion does not necessarily follow from the premises: It can only be concluded to a certain probability. In contrast to probable deduction, necessary deduction always leads from true premises to true conclusions. Thus, necessary deduction corresponds to necessary reasoning. It is worth to note that, according to Peirce, even deductive inquiry is based on experiments too, namely on mental experiments. Roughly speaking, induction is based on many experiments in the real world, and deduction is based on one experiment in the mind.[4]

Finally, besides induction and deduction, *abduction* is a creative generation of a new hypothesis and its provisional adoption. For a hypothesis which is obtained by abduction, its consequences are capable of experimental verification, and if further, new experiments contradict the hypothesis, it will be ruled out.

In induction and abduction, the conclusions are hypothetical, thus it is clear that these modes of reasoning tend to correct themselves. But this applies to deduction as well. Already at the beginning of lecture 4 in [Pei92], Peirce says that '*deductive inquiry, then, has its errors; and it corrects them, too*', and two pages later he concludes '*that inquiry of every type, fully carried out, has the vital power of self-correction and of growth.*' Now we see why Peirce was convinced that mathematical reasoning is such reliable: '*The certainity of mathematical reasoning, however, lies in this, that once an error is suspected, the whole world is speedily in accord about it.*'

2.5 Rational Communication

The last quotation sheds a new light to another important aspect in Peirce's theory of reasoning and knowledge, namely the importance of a rational community (the 'whole word' [of mathematicians] in the quotation above). In Peirce's understanding, knowledge is an collective achievement. It grows by means of communication between human beings, where the results of reasoning are critically observed and discussed. In any moment, the community possesses certain

[4] Mathematical reasoning and diagrammatic reasoning are synonymous for Peirce. In [Eis76], we find an often quoted passage where the use of experiments in diagrammatic reasoning is explained as follows: '*By diagrammatic reasoning, I mean reasoning which constructs a diagram according to a precept expressed in general terms, performs experiments upon this diagram, notes their results, [ldots] and expresses this in general terms.*'

information, obtained from previous experiences, whose results are analyzed by means of reasoning, i.e. deduction, induction, and abduction. Informations are conscious cognitions, and Peirce speaks of '*the cognitions which thus reach us by this infinite series of inductions and hypotheses*' (5.311). This process leads from specific information to more general information, and to the recognition of the reality and truth in the long run. In fact, there is no other way than just described to reach a knowledge of reality: In 5.312, Peirce continues: '*Thus, the very origin of the conception of reality shows that this conception essentially involves the notation of a COMMUNITY, without definite limits, and capable of a definite increase of knowledge*' (emphasis by Peirce). We see that knowledge growths by use of rational communication in a community. It is worth to note that even reasoning carried out by a single person can be understood to be a special kind of rational communication as well. In 5.421, 'What Pragmaticism Is', 1905, Peirce says: '*A person is not absolutely an individual. His thoughts are what 'he is saying to himself', that is, is saying to that other self that is just coming into life in the flow of time*', or in 7.103 he explains: '*In reasoning, one is obliged to think to oneself. In order to recognize what is needful for doing this it is necessary to recognize, first of all, what "oneself" is. One is not twice in precisely the same mental state. One is virtually [...] a somewhat different person, to whom one's present thought has to be communicated.*'

3 Existential Graphs

The discussion so far show up some essential aspects of reasoning: It is self-controlled and self-critical, and it takes places in a community by means of rational communication. For this reason, we need an instrument which allows to explicate and investigate the course of reasoning as best as possible. This is the purpose of EGs, as it is clearly stated by Peirce in 4.248–4.429:

> Now a thorough understanding of mathematical reasoning must be a long stride toward enabling us to find a method of reasoning about this subject as well, very likely, as about other subjects that are not even recognized to be mathematical.
> This, then, is the purpose for which my logical algebras were designed but which, in my opinion, they do not sufficiently fulfill. The present system of existential graphs is far more perfect in that respect, and has already taught me much about mathematical reasoning. [...]
> Our purpose, then, is to study the workings of necessary inference.

This has already been realized by Roberts: He writes in [Rob73] that '*The aim [of EGs] was not to facilitate reasoning, but to facilitate the study of reasoning.*' In the beginning of this paper, it has already been said that Peirce's life-long aim was the investigation of reasoning and knowledge. For him, his EGs turned out to be the right instrument for making necessary reasoning explicit (much better than language), thus the investigation of EGs *is* the investigation of necessary reasoning. From this point of view, the central place of EGs in Peirce's philosophy becomes plausible. Moreover, due to the discussion so far, the design of EGs can be explained as well.

3.1 Patterns of Reasoning and the Transformation Rules

The quotation I have just provided continues as follows: '*What we want, in order to do this, is a method of representing diagrammatically any possible set of premises, this diagram to be such that we can observe the transformation of these premises into the conclusion by a series of steps each of the utmost possible simplicity.*' We have already seen that deductive inquiries are for Peirce mental experiments. In these experiments, we are starting with some facts, and rearrange these facts to obtain new knowledge. First of all, different pieces of information are brought together, that is, they are *colligated*. Then, sometimes, informations are duplicated (or vice versa: redundant information is removed), or some other information which is not needed anymore is erased. These are for Peirce the general figures of reasoning : '*Precisely those three things are all that enter in the Experiment of any Deduction — Colligation, Iteration, Erasure. The rest of the process consists of Observing the result.*' [Pei92]. It is this understanding of reasoning which underlies Peirce's permission rules, i.e. erasure and insertion, iteration and deiteration, and double cut. These rules are the patterns reasoning is composed of.[5]

3.2 The Transformation Rules are not Intended a Calculus

The purpose of the rules is to explicate a reasoning process *a posteriori*, to explain and allow to make mental experiments on diagrams which explicate the premises of the reasoning process, but not to aid the drawing of inferences. In 4.373, he writes:

> The first requisite to understanding this matter is to recognize the purpose of a system of logical symbols. That purpose and end is simply and solely the investigation of the theory of logic, and not at all the construction of a calculus to aid the drawing of inferences. These two purposes are incompatible, for the reason that the system devised for the investigation of logic should be as analytical as possible, breaking up inferences into the greatest possible number of steps, and exhibiting them under the most general categories possible; while a calculus would aim, on the contrary, to reduce the number of processes as much as possible, and to specialize the symbols so as to adapt them to special kinds of inference.

[5] In [Shi02], Shin argues that Peirce's transformation rules are not fully developped in an iconic manner, and she poses the question why Peirce himself did not fully exploit the iconic features of EGs. This might be an answer to her.

Peirce has a very precise understanding of the term 'calculus' (probably based on Leibniz' idea of a 'calculus ratiocinator'). A calculus is not simply a set of (formal) rules acting on a system of symbols. For him, the purpose is essential, and the purpose gives a set of rules its shape. The purpose of a calculus is to support drawing inferences. Thus, the derivations in a calculus are rather short, and the inference steps are rather complicated, because it is the goal to reach the conclusion as fast as possible. A calculus is a *synthetical* tool. In contrast to that, the goal of Peirce's rules is to exhibit the steps of a reasoning process. Thus, the rules are rather simple and correspond the general patterns of reasoning, and the derivations '*dissect the operations of inference into as many distinct steps as possible*' (4.424). Peirce's rules are an *analytical* tool. They allow to discuss and critizise any reasoning best. For this reason, Peirce emphasizes that his system '*is not intended as a calculus, or apparatus by which conclusions can be reached and problems solved with greater facility than by more familiar systems of expression*' (4.424).[6]

3.3 The Universe of Discourse and the Sheet of Insertion

As we have just discussed the purpose and the design of the rules, we will now explore the form and appearance of EGs. I have already qouted 4.431, where Peirce states that necessary reasoning is about assertions in an '*arbitrarily hypothetical universe, a creation of a mind.*' Reasoning can be understood as a rational discourse, and such a discourse takes always place in a specific context, the *universe of discourse*. It is essential for the participants of a discourse to agree on this universe. This is explicated by by Peirce in Logical Tracts. No. 2. 'On Existential Graphs, Euler's Diagrams, and Logical Algebra', MS 492, where he writes: '*The logical universe is that object with which the utterer and the interpreter of any proposition must be well-aquainted and mutually understand each other to be well acquainted, and must understand that all their discourse refers to it.*' EGs are an instrument to make reasoning explicit. The universe of discourse is represented the system of EGs by the sheet of assertion. This function of the sheet of assertion is described in in 4.396 by Peirce as follows: '*It is agreed that a certain sheet, or blackboard, shall, under the name of The Sheet of Assertion, be considered as representing the universe of discourse [...].*'

[6] Ironically, compared to the rules of contemporary calculi for first order logic (like natural deduction), the rules for Peirce's EGs turn out to be rather complex. Moreover, nowadays it is often said that a main advantage of Peirce's rules is that they allow to draw very *short* inferences. For example, in his commentary on Peirce's MS 514, Sowa provides an proof for Leibniz's Praeclarum Theorema (splendid theorem) with Peirce's rules, which needs 7 steps, and writes later on: '*In the Principia Mathematica, which Whitehead and Russell (1910) published 13 years after Peirce discovered his rules, the proof of the Praeclarum Theorema required a total of 43 steps, starting from five non-obvious axioms. One of those axioms was redundant, but the proof of its redundancy was not discovered by the authors or by any of their readers for another 16 years. All that work could have been saved if Whitehead and Russell had read Peirce's writings on existential graphs.*'

Using a sheet of assertion for representing the universe of discourse is no accident, but a consequence of Peirce's purpose –making reasoning explicit– of EGs. This is explained by Peirce in 4.430 as follows:

> What we have to do, therefore, is to form a perfectly consistent method of expressing any assertion diagrammatically. The diagram must then evidently be something that we can see and contemplate. Now what we see appears spread out as upon a sheet. Consequently our diagram must be drawn upon a sheet. We must appropriate a sheet to the purpose, and the diagram drawn or written on the sheet is to express an assertion. We can, then, approximately call this sheet our sheet of assertion.

An empty sheet of assertion represents the very beginning of a discourse, when no assertions so far are made. A diagram represents a proposition, and writing the diagram on the sheet of assertion is to assert it (that is, the corresponding proposition). Peirce had a very broad understanding of the term 'diagram' (see for example [Dau04, Shi02]), so the question arises how the diagrams should look like. As diagrams have to be contemplated, the underlying goal is that '*a diagram ought to be as iconic as possible; that is, it should represent relations by visible relations analogous to them* ' (4.433). This goal induces some design decisions Peirce has made in the development of existential graphs.

3.4 Juxtaposition of Graphs

Peirce continues in 4.433 with an example where two propositions can be taken for true, that is, each of them may be scribed on the sheet of assertion. Let us denote them by P1 and P2. Now it is a self-suggesting idea that *both* P1 *and* P2 may be written on different parts of the sheet of assertion. We then see that P1 *and* P2 are written on the sheet of assertion, and it is very natural to interpret this as the assertion of both P1 *and* P2. Writing two graphs on the sheet of assertion is called *juxtaposing* these graphs, and we have just seen that the juxtaposition of graphs is a highly iconical representation of their conjunction (to be very precisely: the conjunction of the propositions which are represented by the graphs).[7] Note that the juxtaposition of graphs is a commutative and associative operation, thus the commutativity and associativity of conjunction is iconically captured by its representation and has –in contrast to linear forms of logic– not to be covered by rules.

[7] Before Peirce's invention of existential graphs, he worked shortly on a system called *entitative* graphs. In this system, the juxtaposition of two graphs is interpreted as the *disjunction* of the graphs. Peirce realized that this interpretation is counter-intuitive, so he switched to to interpreting the juxtaposition as a conjunction. See [Rob73] for a thorough discussion.

3.5 Lines of Identity

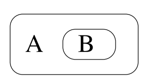
There are several places where Peirce discusses the iconicity of the line of identity. Assume that each of the letters A and B stands for a unary predicate, i.e. we have to complete each of them by an object to obtain a proposition which is false or true (mathematically spoken: A and B are the names of relations with arity 1). Assume we know that both A and B can be completed by the *same* object in order to get a true proposition? In 4.385, Peirce answers as follows:

> A very iconoidal way of representing that there is one quasi-instant [the object] at which both A and B are true will be to connect them with a heavy line drawn in any shape, thus:
>
> 
>
> If this line be broken, thus A— —B, the identity ceases to be asserted.[8]

(A very similar argumentation can be found in 4.442.) In 4.448, he argues that a line of identity is a mixture of a symbol, an index and an icon. Nonetheless, although Peirce does not think that lines of identity are purely iconic, he concludes 4.448 with the following statement: '*The line of identity is, moreover, in the highest degree iconic.*'

3.6 Cuts

In necessary reasoning, Peirce focuses on implications. In the system of existential graphs, a device of two nested cuts, a scroll, can be read as an implication. For example,



is read 'A implies B'. Reading a scroll as implication is usually obtained from the knowledge that cuts represent negation, i.e., the graph is literally read 'it is not true that A holds and B does not hold', which is equivalent to 'A implies B'. In 4.376–4.379, Peirce discusses how implications have to be handled in any logical system, and he draws the conclusion that syntactical devices are needed which allows to separate the premise from the sheet of assertion resp. the conclusion from the the premise, and he argues that this syntactical device negates the part of the implication which is separated by it. Separating a part of a graph which is written on the sheet of assertion is represented on a sheet by a closed line. For this reason, he added the cut as a syntactical devise to graphs, and due to the argument he has provided before, he *concludes* that the cut negates the enclosed subgraph.

[8] The two lines of identity *may* denote distinct objects, but this is not *necessary*, i.e., they are still allowed to denote the same objects as well.

We see that the design, the appearance of EGs is, similarly to the design of the rules, driven by Peirce's purpose to provide an instrument for investigating reasoning.

4 Conclusion

To conclude this paper, it shall be remarked that Peirce himself stresses in 4.424 that his purpose for EGs has not to be confused with other purposes. We have already seen that EGs are not intended as a calculus. Moreover, Peirce stresses that '*this system is not intended to serve as a universal language for mathematicians or other reasoners.*' A universal language is intended to describe only one, i.e., 'the', universe. In a universal language, the signs have a fixed, definite meaning. i.e. there are no different interpretations of a sign.[9] But EGs are about arbitrarily hypothetical universes, and they have to be interpreted in a given universe of discourse (Peirce describes the handling of EGs by means of a communication between a so-called *graphist*, who asserts facts by scribing and manipulating appropriate graphs on the sheet of assertion, and a so-called *grapheus* or *interpreter*[10], who interprets the graphs scribed by the grapheus and checks their validity in the universe of discourse). Thus it is clear that EGs cannot serve as a universal language.

Moreover, although EGs are intended to provide an instrument for the investigation of reasoning, it is important for Peirce that the psychological aspects of reasoning are not taken into account.[11] Finally, Peirce writes that '*although there is a certain fascination about these graphs, and the way they work is pretty enough, yet the system is not intended for a plaything, as logical algebra has sometimes been made.*' After we have elaborated in this paper why Peirce placed his EGs into the very center of his philosophy, this assessment is by no means surprising.

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[9] This was Frege's intention when he invented his Begriffsschrift.

[10] Due to Peirce's understanding that reasoning is established by communication, the graphist and grapheus may be different mental states of the same person.

[11] Susan Haack distinguishes in her book 'philosophy of logics' ([Haa78]) three approaches to logic: strong psychologism, where '*logic is descriptive of mental processes*', weak psychologism, where '*logic is prescriptive of mental processes (it prescribes how we* should *think*', and anti-psychologism, where '*logic has nothing to do with mental processes*', and names Kant, Peirce and Frege as examples for these three approaches, respectively.

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